مدرسة مارية القبطية
للتعليم الثانوي

G12 ADVANCED

- Lesson 5.1
- Lesson name: Antiderivatives

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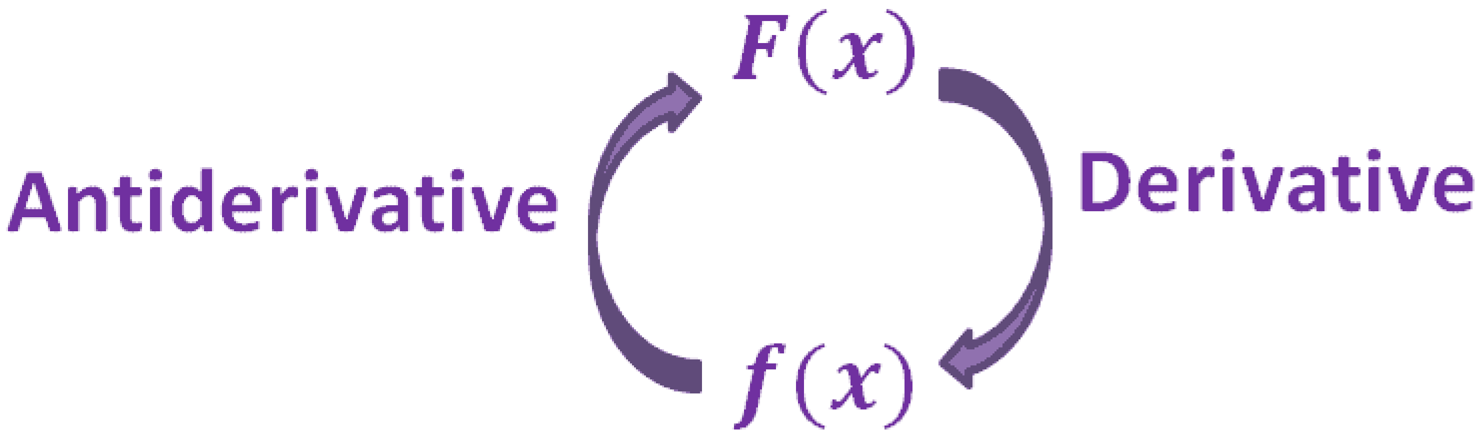
- Find the antiderivative of a given function.
- Understand the notion of indefinite integral as finding an antiderivative.
- Compute straightforward indefinite integrals.

Title: 5.1 Antiderivatives

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

- Antiderivative
- Indefinite Integral
- Integration



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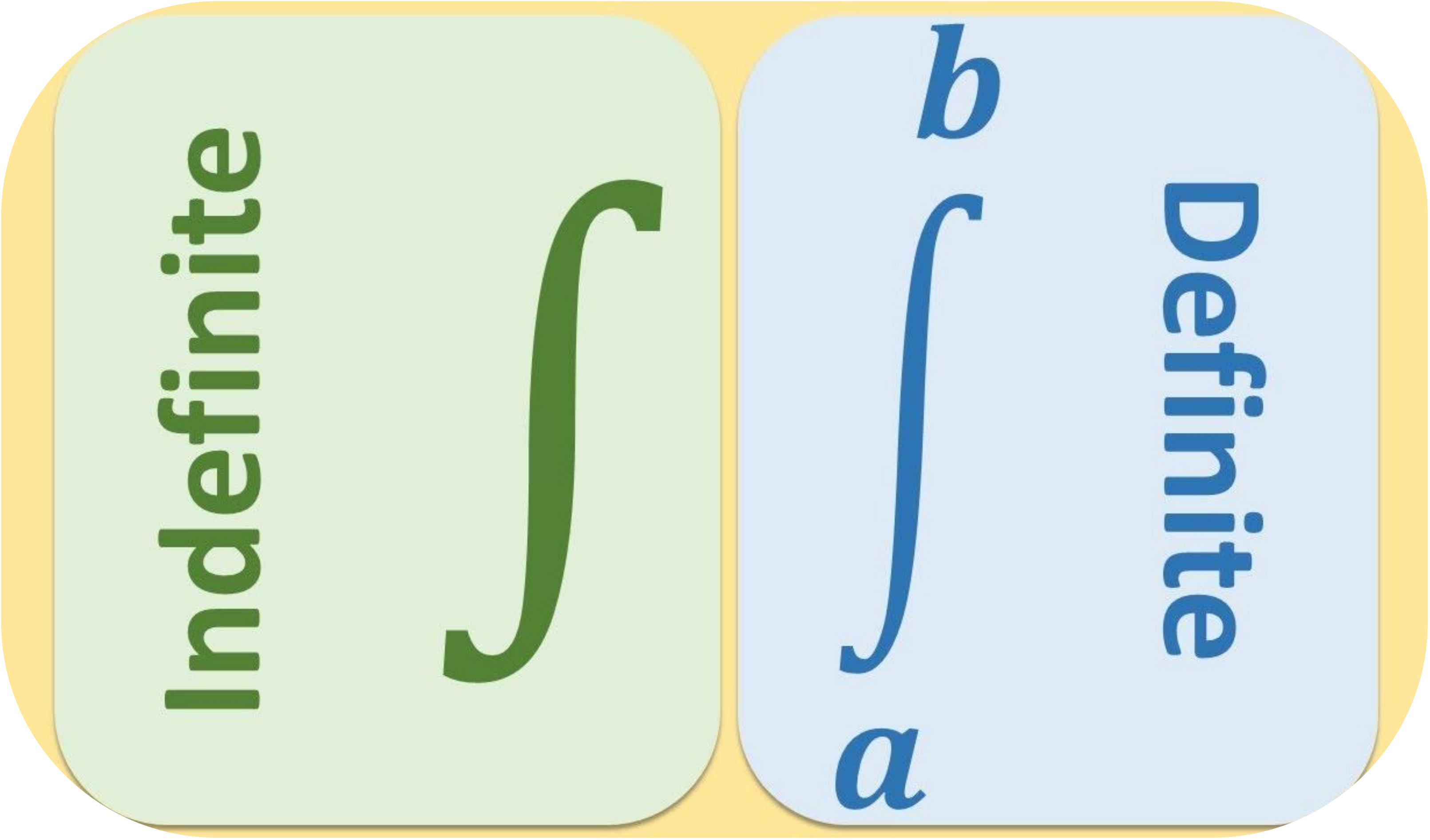

Let F be any antiderivatives of f on an interval I. The **Indefinite Integral** of $f(x)$ (*with respect to* x) on I, is defined by

$$\int f(x)dx = F(x) + c,$$

Where c is an *arbitrary constant* (the **constant of integration**).

Integration: The process of computing the integral

$$\int f(x)\,dx.$$

variable of integration is x

Integrand

DEFINITION 1.1

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$y = x^2$ so $\frac{dy}{dx} = 2x$

$y = x^2 + 3$ so $\frac{dy}{dx} = 2x$

$y = x^2 + 75$ so $\frac{dy}{dx} = 2x$

$y = x^2 + 250$ so $\frac{dy}{dx} = 2x$

$y = x^2 + C$ so $\frac{dy}{dx} = 2x$

Now if we want to INTEGRATE (or antidifferentiate) $2x$

we basically get x^2 but as you can see above, the antiderivative or integral could be $2x$ plus ANY CONSTANT NUMBER C

So if $\frac{dy}{dx} = 2x$ then $y = x^2 + C$

For any power $r \neq -1$,

$$\int x^r dx = \frac{x^{r+1}}{r+1} + c,$$

Here, if $r < -1$, the interval I on which this is defined can be any interval that does not include $x = 0$.

$$\frac{d}{dx}\left[\frac{x^{r+1}}{r+1} + c\right] = (r+1)\frac{x^{r+1-1}}{r+1} = x^r$$

This proves theorem 1.2

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

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Using Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq 1$$

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Evaluate $\int x^{17}dx$

$$\int x^{17}dx$$

$$= \frac{x^{17+1}}{17+1} + c$$

$$= \frac{x^{18}}{18} + c$$

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Evaluate $\int x^3 dx$

$$\int x^3 dx$$

$$= \frac{x^{3+1}}{3+1} + c$$

$$= \frac{x^4}{4} + c$$

Title: 5.1 Antiderivatives

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Exercise

a) $\int u^2 du$ b) $\int x\, dx$ c) $\int dx$ d) $\int m^4 dm$

Solution

a

$$\int u^2 du$$

We are integrating with respect to u
Using Power Rule

$$= \frac{u^{2+1}}{2+1} + c = \frac{u^3}{3} + c$$

c

$$\int dx = \int 1 \cdot dx$$

We are integrating with respect to x
Here the power of x is zero
$1 = x^0$
Using Power Rule

$$= \frac{x^{0+1}}{0+1} + c = \frac{x^1}{1} + c$$
$$= x + c$$

b

$$\int x\, dx$$

We are integrating with respect to x
Using Power Rule

$$= \frac{x^{1+1}}{1+1} + c = \frac{x^2}{2} + c$$

d

$$\int m^4 dm$$

We are integrating with respect to m
Using Power Rule

$$= \frac{m^{4+1}}{4+1} + c = \frac{m^5}{5} + c$$

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The Power Rule with a Negative Exponent

$$\frac{1}{x^{-n}} = x^n$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq 1$$

$$\int \frac{1}{x^3}\,dx$$

Rewrite the integrand

$$= \int x^{-3}\,dx$$

Using Power Rule

$$= \frac{x^{-3+1}}{-3+1} + c$$

$$= \frac{x^{-2}}{-2} + c$$

$$= -\frac{1}{2x^2} + c$$

$$\int x^{-5}\,dx$$

Using Power Rule

$$= \frac{x^{-5+1}}{-5+1} + c$$

$$= \frac{x^{-4}}{-4} + c$$

$$= -\frac{1}{4x^4} + c$$

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The Power Rule with a Negative Exponent

$$\frac{1}{x^{-n}} = x^n$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq 1$$

$$\int \sqrt{x}\ dx$$

Rewrite the integrand In exponent form

$$= \int x^{\frac{1}{2}}\,dx$$

Using Power Rule

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c \quad = \frac{2x^{\frac{3}{2}}}{3} + c$$

$$= \frac{2\sqrt{x^3}}{3} + c$$

$$\int \frac{1}{\sqrt[3]{x}}\,dx$$

Rewrite the integrand In exponent form

$$= \int \frac{1}{x^{\frac{1}{3}}}\,dx \quad = \int x^{-\frac{1}{3}}\,dx$$

Using Power Rule

$$= \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c \quad = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c \quad = \frac{3x^{\frac{2}{3}}}{2} + c$$

$$= \frac{3\sqrt[3]{x^2}}{2} + c$$

Title: 5.1 Antiderivatives

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Derivative	Antiderivative=Integration
$\frac{d}{dx}(F(x)) = f(x)$	$\int f(x)\,dx = F(x) + c$
$\frac{d}{dx}(a) = 0$ $\quad \frac{d}{dx}(ax) = a$	$\int a\,dx = ax + c$
$\frac{d}{dx}(x^2) = 2x$	$\int x\,dx = \frac{x^2}{2} + c$
$\frac{d}{dx}(x^{n+1}) = (n+1)x^n$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad , n \neq 1$

THEOREM 1.2 (Power Rule)

$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}(e^{-x}) = -e^{-x}$	$\int e^{-x}\,dx = -e^{-x} + c$
$\frac{d}{dx}(a^x) = a^x.\ln a$	$\int a^x\,dx = \frac{1}{\ln a}a^x + c$
$\frac{d}{dx}(\ln x) = \frac{1}{x} = x^{-1}$	$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln x + c$

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$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\, dx = \sin x + c$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\, dx = -\cos x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + c$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + c$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + c$

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$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = sin^{-1}x + c$
$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = tan^{-1}x + c$
$\frac{d}{dx}(sec^{-1}x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}}\,dx = sec^{-1}x + c$

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Let $y = \sin x$ ➾ $\frac{dy}{dx} = \frac{d}{dx}[sinx\} = cosx$

We get the corresponding integration formula $\int cosx\, dx = sinx + c$

Let $y = e^x$ ➾ $\frac{dy}{dx} = \frac{d}{dx}[e^x] = e^x$

We get the corresponding integration formula $\int e^x dx = e^x + c$

Let $y = tan^{-1}x$ ➾ $\frac{dy}{dx} = \frac{d}{dx}[tan^{-1}x] = \frac{1}{1+x^2}$

We get the corresponding integration formula $\int \frac{1}{1+x^2} dx = tan^{-1}x + c$

An Indefinite Integral of a Sum and a Difference

$$\int [a.f(x) + b.g(x)]dx = a.\int f(x)dx + b.\int g(x)dx.$$

$$\int (x^3 + 2x - 7)dx$$

Using Theorem 3.1

$$= \int x^3 dx + 2\int x\,dx - \int 7\,dx$$

$$= \frac{x^4}{4} + 2\frac{x^2}{2} - 7x + c$$

$$= \frac{x^4}{4} + x^2 - 7x + c$$

$$\int (mx + a)dx$$

Integrate with respect to x
m and a are constants

Using Theorem 3.1

$$= m\int x\,dx + \int a\,dx$$

$$= m\frac{x^2}{2} + ax + c$$

$$= \frac{mx^2}{2} + ax + c$$

Compute straightforward indefinite integrals.

An Indefinite Integral of a Sum and a Difference

$$\int [a.f(x) + b.g(x)]dx = a.\int f(x)dx + b.\int g(x)dx.$$

$$\int (3\cos x + 4x^8)dx$$

Using Theorem 3.1

$$= 3\int cosx\, dx + 4\int x^8 dx$$

$$= 3sinx \quad + 4\frac{x^9}{9} \quad + c$$

$$= 3sinx + \frac{4}{9}x^9 + c$$

$$\int \left(2x^{-2} + \frac{1}{\sqrt{x}}\right) dx$$

Using Theorem 3.1

$$= 2\int x^{-2}dx \quad + \int \frac{1}{\sqrt{x}}dx$$

Rewrite the integrand to apply the power rule

$$= 2\int x^{-2}dx \; + \int x^{-\frac{1}{2}}\, dx$$

$$= 2\frac{x^{-1}}{-1} \quad + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \quad + c$$

$$= -\frac{2}{x} + 2\sqrt{x} + c$$

Compute straightforward indefinite integrals.

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An Indefinite Integral of a Sum and a Difference

$$\int [a.f(x) + b.g(x)]dx = a.\int f(x)dx + b.\int g(x)dx.$$

$$\int \left(3e^x - \frac{2}{1+x^2}\right) dx$$

Using Theorem 3.1

$$= 3\int e^x dx \; -2\int \frac{1}{1+x^2} dx$$

$$\int \frac{1}{1+x^2} dx = tan^{-1}x + c$$

$$= \; 3e^x - \; 2\,tan^{-1}x + c$$

Compute straightforward indefinite integrals.

$$\int \left(3\sqrt{x} - \frac{1}{x^4}\right) dx$$

Using Theorem 3.1

$$= 3\int \sqrt{x}\, dx \; - \int \frac{1}{x^4} dx$$

Rewrite the integrand to apply the power rule

$$= 3\int x^{\frac{1}{2}} dx \; - \int x^{-4} dx$$

$$= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{-3}}{-3} + c$$

$$= 3\frac{2x^{\frac{3}{2}}}{3} + \frac{x^{-3}}{3} + c$$

$$= 2\sqrt{x^3} + \frac{1}{3x^3} + c$$

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An Indefinite Integral of a Sum and a Difference

$$\int [a.f(x) + b.g(x)]dx = a.\int f(x)dx + b.\int g(x)dx.$$

$$\int (4x - 2e^x)dx$$

Using Theorem 3.1

$$= 4\int x\ dx\ -2\int e^x dx$$

$$= 4\frac{x^2}{2} - 2e^x\ +c$$

$$= 2x^2 - 2e^x + c$$

$$\int (3cosx - sinx)dx$$

Using Theorem 3.1

$$= 3\int cosx\ dx\ - \int sinx\ dx$$

$$= 3sinx\ \ - -cosx\ \ +c$$

$$= 3sinx + cosx + c$$

Compute straightforward indefinite integrals.

Finding a Function Given Its Derivative

$$\int 2\, secx\, tanx\, dx$$ *Using Theorem 3.1*

$$= 2\int secx\, tanx\, dx$$

$$= 2\, sec\, x\ + c$$

a

$$\int \frac{4}{\sqrt{1-x^2}}\, dx$$ *Using Theorem 3.1*

$$= 4\int \frac{1}{\sqrt{1-x^2}}\, dx \qquad = 4sin^{-1}x + c$$

b

$$\int 5sec^2x\, dx$$ *Using Theorem 3.1*

$$= 5\int sec^2x\, dx \qquad = 5tan\, x + c$$

Compute straightforward indefinite integrals.

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Simplifying Integrand Before Finding the Antiderivative $x^n . x^m = x^{n+m}$

$$\int \frac{x^{1/3}-3}{x^{2/3}}dx$$

Distribute the numerator on the denominator

$$\int\left(\frac{x^{1/3}}{x^{2/3}}-\frac{3}{x^{2/3}}\right)dx = \int\left(x^{\frac{1}{3}-\frac{2}{3}}-3x^{-\frac{2}{3}}\right)dx$$

$$\int\left(x^{-1/3}-3x^{-2/3}\right)dx$$

Using Theorem 1.3

$$\int x^{-1/3}dx - 3\int x^{-2/3}dx$$

$$=\frac{x^{2/3}}{\frac{2}{3}}-3\frac{x^{1/3}}{\frac{1}{3}}+c \qquad =\frac{3x^{2/3}}{2}-3\frac{3x^{1/3}}{1}+c$$

$$=\frac{3\sqrt[3]{x^2}}{2}-9\sqrt[3]{x}+c$$

$$\int \frac{x+2x^{3/4}}{x^{5/4}}dx$$

Distribute the numerator on the denominator

$$\int\left(\frac{x}{x^{5/4}}+\frac{2x^{3/4}}{x^{5/4}}\right)dx = \int\left(x^{1-\frac{5}{4}}+2x^{\frac{3}{4}-\frac{5}{4}}\right)dx$$

$$\int\left(x^{-1/4}+2x^{-2/4}\right)dx$$

Using Theorem 1.3

$$\int x^{-1/4}dx + 2\int x^{-2/4}dx$$

$$=\frac{x^{3/4}}{\frac{3}{4}}+2\frac{x^{2/4}}{\frac{2}{4}}+c \qquad =\frac{4x^{3/4}}{3}+2\frac{4x^{2/4}}{2}+c$$

$$=\frac{4\sqrt[4]{x^3}}{3}+4\sqrt{x}+c$$

Title: 5.1 Antiderivatives

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

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Simplifying Integrand Before Finding the Antiderivative $x^n . x^m = x^{n+m}$

$$\int x^{1/4}(x^{5/4} - 4)\, dx$$ *Multiply terms of integrand*

$$\int \left(x^{\frac{1}{4}+\frac{5}{4}} - 4x^{1/4}\right)dx = \int \left(x^{6/4} - 4x^{1/4}\right)dx$$ *Simplify*

$$\int \left(x^{3/2} - 4x^{1/4}\right)dx$$ *Using Theorem 1.3*

$$\int x^{3/2}dx - 4\int x^{1/4}dx$$

$$= \frac{x^{5/2}}{\frac{5}{2}} - 4\frac{x^{5/4}}{\frac{5}{4}} + c \qquad = \frac{2x^{5/2}}{5} - 4\frac{4x^{5/4}}{5} + c$$

$$= \frac{2\sqrt{x^5}}{5} - \frac{16\sqrt[4]{x^5}}{5} + c$$

$$\int x^{2/3}(x^{-4/3} - 3)\, dx$$ *Multiply terms of integrand*

$$\int \left(x^{\frac{2}{3}+\frac{-4}{3}} - 3x^{2/3}\right)dx = \int \left(x^{-2/3} - 3x^{2/3}\right)dx$$

Using Theorem 1.3

$$\int x^{-2/3}dx - 3\int x^{2/3}dx$$

$$= \frac{x^{1/3}}{\frac{1}{3}} - 3\frac{x^{5/3}}{\frac{5}{3}} + c \qquad = \frac{3x^{1/3}}{1} - 3\frac{3x^{5/3}}{5} + c$$

$$= 3\sqrt[3]{x} - \frac{9\sqrt[3]{x^5}}{5} + c$$

Compute straightforward indefinite integrals.

Simplifying Integrand Before Finding the Antiderivative

$$\int 4\frac{\cos x}{\sin^2 x}dx$$ *Rewrite the integrand*

$$= \int 4\frac{\cos x}{\sin x \cdot \sin x}dx = \int 4\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}dx$$ *Simplify*

$$= \int 4\cot x \cdot \csc x\, dx$$ *Using Theorem 1.3*

$$= 4\int \csc x \cot x\, dx$$

$$= -4\csc x + c$$

$$\int \frac{e^x + 3}{e^x}dx$$ *Distribute the numerator on the denominator*

$$= \int \left(\frac{e^x}{e^x} + \frac{3}{e^x}\right)dx$$ *Simplify*

$$= \int (1 + 3e^{-x})dx$$ *Using Theorem 1.3*

$$= \int 1\, dx + 3\int e^{-x}dx$$

$$= x + 3(-e^{-x}) + c \quad = x - 3e^{-x} + c$$

Compute straightforward indefinite integrals.

Simplifying Integrand Before Finding the Antiderivative

$$\int \frac{3}{4x^2+4}\,dx$$ *Rewrite the integrand*

$$= \int \frac{3}{4(x^2+1)}\,dx$$ *Using Theorem 1.3*

$$= \frac{3}{4}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{3}{4}\tan^{-1}x + c$$

$$\int \left(2\cos x - \sqrt{e^{2x}}\right)dx$$ *Rewrite the integrand*

$$= \int \left(2\cos x - (e^{2x})^{\frac{1}{2}}\right)dx$$ *Simplify*

$$= \int (2\cos x - e^{x})\,dx$$ *Using Theorem 1.3*

$$= 2\int \cos x\,dx - \int e^{x}\,dx$$

$$= 2\sin x - e^{x} + c$$

Compute straightforward indefinite integrals.

For $x \neq 0$,

$$\frac{d}{dx}\ln|x| = \frac{1}{x}.$$

Using Chain Rule and Theorem 1.4 we get:

$$\frac{d}{dx}[ln|f(x)|] = \frac{\frac{d}{dx}[f(x)]}{f(x)} = \frac{f'(x)}{f(x)}$$

Where $f(x) \neq 0$

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The Derivative of the Log of an Absolute Value

COROLLARY 1.1	COROLLARY 1.2
In any interval not containing 0, $\int \frac{1}{x}\,dx = \ln|x| + c$	In any interval not containing 0, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

The Indefinite Integral of a Fraction of the Form $\frac{f'(x)}{f(x)}$

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The Indefinite Integral of a Fraction of the Form $\frac{f'(x)}{f(x)}$

$$\int \left(\frac{3}{x} - 2\cos x\right) dx$$

Using Theorem 1.3

$$= 3\int \frac{1}{x} dx - 2\int \cos x \, dx$$

Using Corollary 1.1

$$= 3\ln|x| - 2\sin x + c$$

$$\int \left(\frac{3}{x^2+1} + 7x^{-1}\right) dx = \int \left(\frac{3}{x^2+1} + \frac{7}{x}\right) dx$$

Using Theorem 1.3

$$= 3\int \frac{1}{x^2+1} dx + 7\int \frac{1}{x} dx$$

Using Corollary 1.1

$$= 3\tan^{-1}x + 7\ln|x| + c$$

The Indefinite Integral of a Fraction of the Form $\frac{f'(x)}{f(x)}$

$$\int \frac{sec^2 x}{tan\, x}\, dx$$

The numerator is the derivative of Denominator

$$\frac{d}{dx}[tanx] = sec^2 x$$

$$= ln|tanx| + c$$

Remember

$$\int \frac{f'(x)}{f(x)}\, dx = ln|f(x)| + c$$

$$\int \frac{e^x}{e^x + 3}\, dx$$

The numerator is the derivative of Denominator

$$\frac{d}{dx}[e^x + 3] = e^x$$

$$= ln|e^x + 3| + c$$

Remember

$$\int \frac{f'(x)}{f(x)}\, dx = ln|f(x)| + c$$

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The Indefinite Integral of a Fraction of the Form $\frac{f'(x)}{f(x)}$

$$\int \frac{\cos x}{\sin x}\,dx$$

The numerator is the derivative of Denominator

$$\frac{d}{dx}[\sin x] = \cos x$$

$$= \ln|\sin x| + c$$

Remember

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int \frac{4x}{x^2+4}\,dx$$

Rewrite the integrand

$$= \int \frac{2(2x)}{x^2+4}\,dx$$

Using Theorem 1.3

$$= 2\int \frac{2x}{x^2+4}\,dx$$

The numerator is the derivative of Denominator

$$\frac{d}{dx}[x^2+4] = 2x$$

$$= 2\ln|x^2+4| + c$$

Remember

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

The Indefinite Integral of a Tangent and Cotangent Functions

$$\int tanx\, dx$$

Rewrite the integrand

$$= \int \frac{sinx}{cosx} dx \quad = \int \frac{- -sinx}{cosx} dx$$

Using Theorem 1.3

$$= -\int \frac{-sinx}{cosx} dx$$

The numerator is the derivative of Denominator

$$\frac{d}{dx}[cosx] = -sinx$$

Using Corollary 1.2

$$\int tanx\, dx \quad = -ln|cosx| + c$$

$$\int tanx\, dx \quad = ln|(cos\, x)^{-1}| \quad + c$$

$$\int tanx\, dx = ln\left|\frac{1}{cos\, x}\right| + c$$

$$\int tanx\, dx = ln|secx| + c$$

$$\int cotx\, dx$$

Rewrite the integrand

$$= \int \frac{cosx}{sinx} dx$$

The numerator is the derivative of Denominator

$$\frac{d}{dx}[sinx] = cosx$$

$$\int cotx\, dx \quad = ln|sin\, x| \quad + c$$

Using Corollary 1.2

Compute straightforward indefinite integrals.

Identifying Integrals That We Cannot Yet Evaluate

A $\int secx\, dx$

Solution $\int secx\, dx$ *Cannot be evaluated yet by the rules we learnt so far*

B $\int xsin2x\, dx$

Solution $\int xsin2x\, dx$ *Cannot be evaluated yet by the rules we learnt so far*

C $\int \sqrt{x^3+4}\, dx$

Solution $\int \sqrt{x^3+4}\, dx$

Cannot be evaluated yet by the rules we learnt so far

Find all functions satisfying the given conditions

Q35 Page: 330

Find the function $f(x)$ **satisfying** the given **conditions**: $f'(x) = 3e^x + x, \quad f(0) = 4$

$$f'(x) = 3e^x + x$$

$$f(x) = \int f'(x)\,dx = \int (3e^x + x)dx$$

$$f(x) = 3\int e^x\,dx + \int x\,dx$$

$$f(x) = 3e^x + \frac{x^2}{2} + c$$

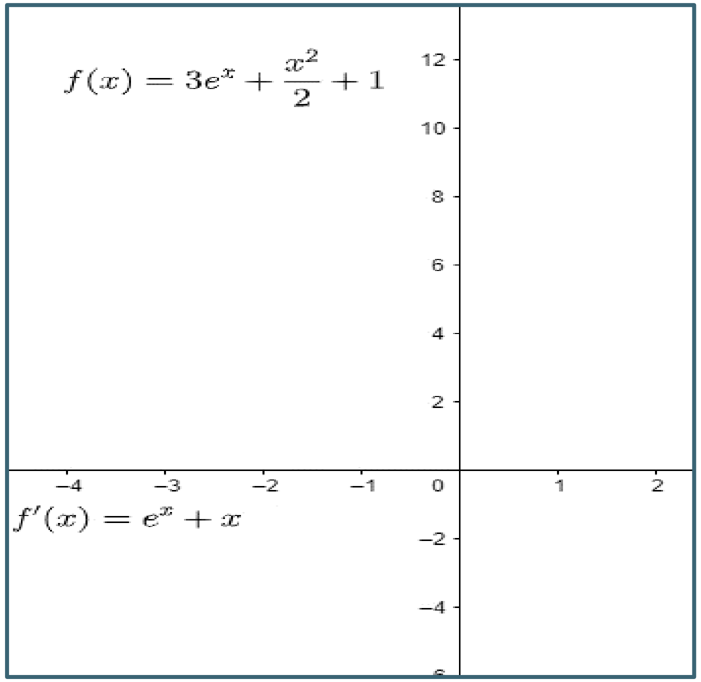
To find the constant of integration c

Use the initial condition $f(0) = 4$

$$f(0) = 3e^{(0)} + \frac{(0)^2}{2} + c = 4$$

$\Rightarrow 3(1) + 0 + c = 4 \Rightarrow c = 1$

$$f(x) = 3e^x + \frac{x^2}{2} + 1$$



Title: 5.1 Antiderivatives

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Q36 Page: 330 **Find** the function $f(x)$ **satisfying** the given **conditions**: $f'(x) = 4cosx, \quad f(0) = 3$

$$f'(x) = 4cosx$$

$$f(x) = \int f'(x)\,dx = \int 4cosx\,dx$$

$$f(x) = 4\int \cos x\,dx$$

$$f(x) = 4\sin x + c$$

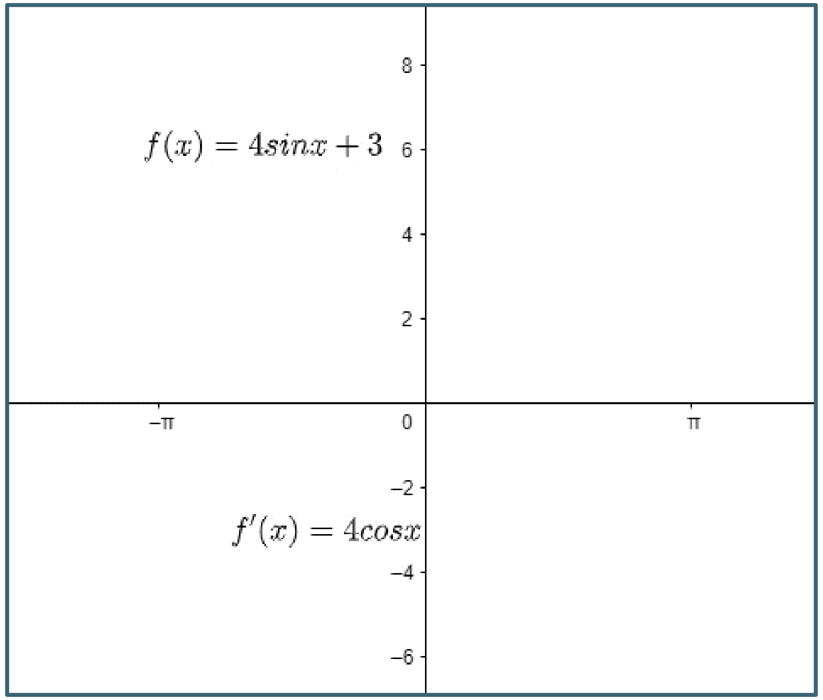
To find the constant of integration c

Use the initial condition $f(0) = 3$

$$f(0) = 4\sin(0) + c = 3$$

$\Rightarrow 4(0) + c = 3 \quad \Rightarrow \quad c = 3$

$$f(x) = 4\sin x + 3$$



Compute straightforward indefinite integrals.

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Find all functions satisfying the given conditions

$$f'''(x) = 4 - \frac{2}{x^3}$$

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Finding a Function Given Its Derivative

$$f'''(x) = 4 - \frac{2}{x^3}$$

$$f''(x) = \int f'''(x)dx = \int \left(4 - \frac{2}{x^3}\right) dx$$

$$f''(x) = \int (4 - 2x^{-3})dx \quad = \int 4\, dx - 2\int x^{-3}dx$$

$$f''(x) = 4x - 2\frac{x^{-2}}{-2} + c_1 \quad = 4x + x^{-2} + c_1$$

There is no initial condition to evaluate c_1

$$f'(x) = \int f''(x)dx = \int (4x + x^{-2} + c_1)dx$$

$$f'(x) = 4\int x\, dx + \int x^{-2}dx + \int c_1 dx$$

$$f'(x) = 4\frac{x^2}{2} + \frac{x^{-1}}{-1} + c_1x + c_2$$

$$f'(x) = 2x^2 - \frac{1}{x} + c_1x + c_2$$

$$f(x) = \int f'(x)\, dx = \int \left(2x^2 - \frac{1}{x} + c_1x + c_2\right) dx$$

$$f(x) = 2\int x^2 dx - \int \frac{1}{x}dx + c_1\int x\, dx + \int c_2 dx$$

$$f(x) = 2\frac{x^3}{3} - ln|x| + c_1\frac{x^2}{2} + c_2x + c_3$$

This represents a family of functions , as we can't evaluate integration constants.

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Exercise **Find** a function $f(x)$ such that the **point** $(-1,1)$ is **on** the graph of $y = f(x)$, the **slope of the tangent** line **at** $(-1,1)$ **is** 2 and $f''(x) = 6x + 4$.

Q52 Page: 330

Solution

Given: $f(-1) = 1$

$f'(-1) = 2$ *Slope of the tangent at $x = -1$*

$f''(x) = 6x + 4$

$$f'(x) = \int f''(x)dx = \int (6x+4)dx$$

$$f'(x) = 6\int x\,dx + \int 4\,dx$$

$$f'(x) = 6\frac{x^2}{2} + 4x + c_1 = 3x^2 + 4x + c_1$$

To evaluate c_1 use the initial condition

$f'(-1) = 2$

$f'(-1) = 3(-1)^2 + 4(-1) + c_1 = 2$

$\Rightarrow 3 - 4 + c_1 = 2 \Rightarrow -1 + c_1 = 2 \Rightarrow c_1 = 3$

$$f'(x) = 3x^2 + 4x + 3$$

$$f(x) = \int f'(x)\,dx = \int (3x^2 + 4x + 3)dx$$

$$f(x) = 3\int x^2 dx + 4\int x\,dx + \int 3\,dx$$

$$f(x) = 3\frac{x^3}{3} + 4\frac{x^2}{2} + 3x + c_2 = x^3 + 2x^2 + 3x + c_2$$

To evaluate c_2 use the initial condition

$f(-1) = 1$

$f(-1) = (-1)^3 + 2(-1)^2 + 3(-1) + c_2 = 1$

$\Rightarrow -1 + 2 - 3 + c_2 = 1 \Rightarrow -2 + c_2 = 1 \Rightarrow c_2 = 3$

$$\boxed{f(x) = x^3 + 2x^2 + 3x + 3}$$

Position, Velocity and Acceleration

We have studied :

Position $\xrightarrow{\frac{d}{dt}}$ *Velocity* $\xrightarrow{\frac{d}{dt}}$ *Acceleration*

Given $s(t) \xrightarrow{\frac{d}{dt}} \frac{d}{dt}[s(t)] = v(t) \xrightarrow{\frac{d}{dt}} \frac{d}{dt}[v(t)] = a(t)$

To reverse that:

Acceleration $\xrightarrow{\int}$ *Velocity* $\xrightarrow{\int}$ *Position*

If we start with $a(t) \xrightarrow{\int} \int a(t)dt = v(t) \xrightarrow{\int} \int v(t)dt = s(t)$

Initial conditions must be given to find integration constants

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Example

If an object 's downward **acceleration** is given by $y''(t) = -9.8\, m/s^2$, **find** the **position function** $y(t)$. Assume that the **initial velocity is** $y'(0) = -30\, m/s$ and the **initial position is** $y(0) = 30,000\, m$.

1.12 Page: 328

Solution

Acceleration: $y''(t) = -9.8\, m/s^2$

Velocity:

$$v(t) = y'(t) = \int y''(t)dt$$

$$v(t) = y'(t) = \int -9.8\, dt$$

$$v(t) = y'(t) = -9.8t + c$$

Using the initial velocity $y'(0) = -30\, m/s$

$$v(0) = y'(0) = -9.8(0) + c = -30$$

$$\Rightarrow 0 + c = -30 \quad \Rightarrow c = -30$$

$$v(t) = y'(t) = -9.8t - 30$$

Position:

$$y(t) = \int y'(t)dt$$

$$y(t) = \int (-9.8t - 30)dt$$

$$y(t) = -9.8 \int t\, dt - \int 30\, dt$$

$$y(t) = -9.8\frac{t^2}{2} - 30t + c$$

$$y(t) = -4.9t^2 - 30t + c$$

Using the initial position $y(0) = 30,000\, m$

$$y(0) = -4.9(0)^2 - 30(0) + c = 30,000$$

$$\Rightarrow 0 - 0 + c = 30,000 \quad \Rightarrow c = 30,000$$

Position Function

$$\boxed{y(t) = -4.9t^2 - 30t + 30,000}$$

Title: 5.1 Antiderivatives

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Exercise

Q45 Page: 330

Determine the **position function** if the **velocity** function is $v(t) = 3 - 12t\ m/s$ and the **initial position** is $s(0) = 3\ m$.

Solution

Given: *Velocity Function:* $v(t) = 3 - 12t\ m/s$

Initial Position: $s(0) = 3\ m$

Velocity: $v(t) = 3 - 12t\ m/s$

Position:

$$s(t) = \int v(t)\,dt$$

$$s(t) = \int (3 - 12t)dt$$

$$s(t) = \int 3\,dt - 12\int t\,dt$$

$$s(t) = 3t - 12\frac{t^2}{2} + c$$

$$s(t) = 3t - 6t^2 + c$$

Using the initial position $s(0) = 3\ m$

$$s(0) = 3(0) - 6(0)^2 + c = 3$$

$$\Rightarrow c = 3$$

Position Function

$$s(t) = -6t^2 + 3t + 3$$

Exercise

Q47 Page: 330

Determine the **position function** if the **acceleration function is** $a(t) = 3\sin t + 1$, the **initial velocity** is $v(0) = 0$ and the **initial position is** $s(0) = 4$.

Solution

Acceleration: $a(t) = 3\sin t + 1$

Velocity:

$$v(t) = \int a(t)\,dt = \int (3\sin t + 1)dt$$

$$v(t) = 3\int \sin t\,dt + \int 1\,dt$$

$$v(t) = -3\cos t + t + c$$

Using the initial velocity $v(0) = 0$

$$v(0) = -3\cos(0) + (0) + c = 0$$

$\Rightarrow -3(1) + (0) + c = 0 \quad \Rightarrow \quad c = 3$

$$v(t) = -3\cos t + t + 3$$

Position:

$$s(t) = \int v(t)\,dt$$

$$s(t) = \int (-3\cos t + t + 3)dt$$

$$s(t) = -3\int \cos t\,dt + \int t\,dt + \int 3\,dt$$

$$s(t) = -3\sin t + \frac{t^2}{2} + 3t + c$$

Using the initial position $s(0) = 4$

$$s(0) = -3\sin(0) + \frac{(0)^2}{2} + 3(0) + c = 4$$

$\Rightarrow -3(0) + 0 + 0 + c = 4 \quad \Rightarrow \quad c = 4$

Position Function

$$s(t) = -3\sin t + \frac{t^2}{2} + 3t + 4$$

Estimating Distance and Acceleration

The table below shows the **velocity of a falling object** at different times. For each time interval, **estimate** the **distance fallen** and the **acceleration**.

Q70 Page: 331

$t(s)$	0	1.0	2.0	3.0	4.0
$v(t)(m/s)$	0.0	−9.8	−18.6	−24.9	−28.5

Solution

time interval [0,1]

To estimate acceleration in each interval

$a(t) = v'(t)$

Compute the slope of the tangent on each interval (here we assume that the acceleration is constant in each interval)

$$a(t) \approx \frac{v(1) - v(0)}{1 - 0} = \frac{-9.8 - 0}{1 - 0} = -9.8\, m/s^2$$

To estimate distance fallen in each interval

$$Distance = velocity \times time$$

Compute the average velocity on each interval then multiply it by time

$$v_{av} = \frac{v(1) + v(0)}{2} = \frac{-9.8 + 0}{2} = -4.9 m/s$$

$$position\ change = avg\ velocity * change\ in\ time$$

$$-4.9 \times 1 = -4.9 m$$

−ve sign indicates direction(downward)

$$Distance\ fallen = 4.9 m$$

Lesson Objective: Find antiderivatives of given fn by understanding the indefinite integral as the process of finding an antiderivative.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

The table below shows the **velocity of a falling object** at different times.
For each time interval, **estimate** the **distance fallen** and the **acceleration**.

Q70 Page: 331

$t(s)$	0	1.0	2.0	3.0	4.0
$v(t)(m/s)$	0.0	-9.8	-18.6	-24.9	-28.5

Acceleration and distance in each time interval

	$Acceleration\,(m/s^2)$	$Average\,Velocity\,(m/s)$	$Position\ change(m)$	$Distance(m)$
$[0,1]$	$a(t)=\frac{v(1)-v(0)}{1-0}=\frac{-9.8-0}{1-0}$ $=-9.8$	$v_{av}=\frac{-9.8+0}{2}=-4.9$	$-4.9\times 1=-4.9$	4.9
$[1,2]$	$a(t)=\frac{-18.6+9.8}{2-1}=-8.8$	$v_{av}=\frac{-18.6+(-9.8)}{2}=-14.2$	$-14.2\times 1=-14.2$	14.2
$[2,3]$	$a(t)=\frac{-24.9+18.6}{3-2}=-6.3$	$v_{av}=\frac{-24.9+(-18.6)}{2}=-21.75$	$-21.75\times 1=-21.75$	21.75
$[3,4]$	$a(t)=\frac{-28.5+24.9}{4-3}=-3.6$	$v_{av}=\frac{-28.5+(-24.9)}{2}=-26.7$	$-26.7\times 1=-26.7$	26.7

Thank You