Unit-5
Integration

مدرسة مارية القبطية
للتعليم الثانوي

G12 ADVANCED

- **Lesson no. 2**
- **Lesson name: Sums and Sigma Notation**

Done By

Ms. RASIYA FS
Mathematics Department
Mareya Al Qebteya Girls School-Dubai

Title:5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

- **Use the sigma notation to compute basic summation.**

Title:5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Sigma Notation is a concise and convenient way to represent long sums.

For example,

$1 + 2 + 3 + 4 + 5$ (Sum of the first five whole numbers)

$1 + 4 + 9 + 16 + 25 + 36$ (Sum of the squares of first six whole numbers

There is an obvious pattern for numbers involved

If we take a sequence of numbers $a_1, a_2, a_3, \dots, a_n$

then we can write the sum of these numbers(series) as $a_1 + a_2 + a_3 + \cdots + a_n$

Sigma Notation

A shorter way of writing this:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

- Upper limit (Last value of i) → n
- General term of the sequence → a_i
- Greek letter "Sigma" Summation → $\sum$
- Index of Summation → i
- Lower limit (First value of i) → 1

Sigma Notation

Example Write in summation notation : $a)\ \sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{10}$

$b)\ 3^3+4^3+5^3+\cdots+45^3$

2.1 Page: 333

a) $\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{10}$

We have the sum of the square roots of the integers from 1 *to* 10

$$a_i = \sqrt{i}$$

i *from* 1 *to* 10

$i = 1,2,3,\ldots\ldots,10$

$$\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{10} = \sum_{i=1}^{10}\sqrt{i}$$

b) $3^3+4^3+5^3+\cdots+45^3$

We have the sum of the cubes of the integers from 3 *to* 45

$$a_i = i^3$$

i *from* 3 *to* 45

$i = 3,4,5,\ldots\ldots,45$

$$3^3+4^3+5^3+\cdots+45^3 = \sum_{i=3}^{45} i^3$$

Exercise Translate into summation notation $Q1)\ 2(1)^2 + 2(2)^2 + 2(3)^2 + \cdots + 2(14)^2$

Q1, Q2 Page: 337 $Q2)\ \sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \cdots + \sqrt{15-1}$

Q1) $2(1)^2 + 2(2)^2 + 2(3)^2 + \cdots + 2(14)^2$

We have the sum of the double of squares of the integers from 1 *to* 14

$$a_i = 2(i)^2$$

i *from* 1 *to* 14

$i = 1,2,3, \dots\dots, 14$

$$2(1)^2 + 2(2)^2 + 2(3)^2 + \cdots + 2(14)^2 = \sum_{i=1}^{14} 2(i)^2$$

Q2) $\sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \cdots + \sqrt{15-1}$

We have the sum of the square roots of the integers from 2 *to* 15 minus one

$$a_i = \sqrt{i-1}$$

i *from* 2 *to* 15

$i = 2,3,4, \dots\dots, 15$

$$\sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \cdots + \sqrt{15-1} = \sum_{i=2}^{15} \sqrt{i-1}$$

It also can be written

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{14} = \sum_{i=1}^{14} \sqrt{i}$$

Title: 5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Example Write in summation notation : The sum of the first 200 odd positive integers. 2.2 Page: 333

Odd integers : $a_i = 2i - 1$

$i = 1,2,3, \dots\dots, 200$

$$1 + 3 + 5 + 7 + 9 + \cdots + 399 = \sum_{i=1}^{200}(2i - 1)$$

Number of terms = upper limit − lower limit + 1

$= (200 - 1 + 1) = 200$

$(2(1) - 1) \quad +(2(2) - 1) \quad +(2(3) - 1) \quad + \cdots + (2(200) - 1)$

$1 \quad + 3 \quad + 5 \quad + \cdots + 399$

Odd integers : $a_i = 2i + 1$

$i = 0,1,2,3, \dots\dots\dots, 199$

$$1 + 3 + 5 + 7 + 9 + \cdots + 399 = \sum_{i=0}^{199}(2i + 1)$$

Number of terms = upper limit − lower limit + 1

$= (199 - 0 + 1) = 200$

$(2(0) + 1) \quad +(2(1) + 1) \quad +(2(2) + 1) \quad + \cdots + (2(199) + 1)$

$1 \quad + 3 \quad + 5 \quad + \cdots + 399$

Title:5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Example Write out all terms and compute the sums

2.3 Page: 333

a) $\sum_{i=1}^{8}(2i+1)$

Solution $\sum_{i=1}^{8}(2i+1) = (2(1)+1)+(2(2)+1)+(2(3)+1)+(2(4)+1)+(2(5)+1)+(2(6)+1)+(2(7)+1)+(2(8)+1)$

$= 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 80$

b) $\sum_{i=2}^{6} sin(2\pi i)$

Solution $\sum_{i=2}^{6} sin(2\pi i) = sin(2\pi(2)) + sin(2\pi(3)) + sin(2\pi(3)) + sin(2\pi(4)) + sin(2\pi(5)) + sin(2\pi(6))$

$= sin4\pi + sin6\pi + sin8\pi + sin10\pi + sin12\pi = 0$

c)

Solution $\sum_{i=4}^{10} 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 = 5(7) = 35$

Exercise Write out all terms and compute the sums

Q5, Q7 Page: 337

Q5 $\sum_{i=1}^{6} 3i^2$

Solution $\sum_{i=1}^{6} 3i^2 \quad 3(1)^2 + 3(2)^2 + 3(3)^2 + 3(4)^2 + 3(5)^2 + 3(6)^2$

$= 3 + 12 + 27 + 48 + 75 + 108 = 273$

Q7 $\sum_{i=6}^{10} (4i + 2)$

Solution $\sum_{i=6}^{10} (4i + 2) = 4(6) + 2 + 4(7) + 2 + 4(8) + 2 + 4(9) + 2 + 4(10) + 2$

$= 26 + 30 + 34 + 38 + 42 = 170$

Lesson Objective: **Use the sigma notation to compute basic summation.**

If n is any positive integer and c is any constant, then

$$(i)\ \sum_{i=1}^{n} c = c(n-1+1)$$

Sum of constants

$$(ii)\ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Sum of the first n positive integers

$$(iii)\ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of the squares of the first n positive integers

For any constants c and d,

$$\sum_{i=1}^{n}(ca_i + db_i) = c\sum_{i=1}^{n} a_i + d\sum_{i=1}^{n} b_i$$

THEOREM 2.2

Title: 5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Example

Compute: $a) \sum_{i=1}^{8}(2i+1)$ $\quad b) \sum_{i=1}^{800}(2i+1)$

2.4 Page: 335

a) $\sum_{i=1}^{8}(2i+1)$ *Using Theorem 2.2*

$$\sum_{i=1}^{8}(2i+1) = 2\sum_{i=1}^{8} i + \sum_{i=1}^{8} 1$$ *Using Theorem 2.1*

$$= 2\frac{8(8+1)}{2} + 1(8)$$

$$= 8(9) + 8$$

$$= 72 + 8 = 80$$

b) $\sum_{i=1}^{800}(2i+1)$ *Using Theorem 2.2*

$$\sum_{i=1}^{800}(2i+1) = 2\sum_{i=1}^{800} i + \sum_{i=1}^{800} 1$$ *Using Theorem 2.1*

$$= 2\frac{800(800+1)}{2} + 1(800)$$

$$= 800(801) + 800$$

$$= 640,800 + 800 = 641,600$$

Title: 5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Exercise Q9, Q12 Page: 337

Use summation rules to compute the sum:

$$Q9)\ \sum_{i=1}^{70}(3i-1) \qquad Q12)\ \sum_{i=1}^{50}(8-i)$$

Q9) $\sum_{i=1}^{70}(3i-1)$ *Using Theorem 2.2*

$$\sum_{i=1}^{70}(3i-1) = 3\sum_{i=1}^{70} i - \sum_{i=1}^{70} 1$$ *Using Theorem 2.1*

$$= 3\,\frac{70(70+1)}{2} - 1(70)$$

$$= 3(35)(71) - 70$$

$$= 7455 - 70 = 7385$$

Q12) $\sum_{i=1}^{50}(8-i)$ *Using Theorem 2.2*

$$\sum_{i=1}^{50}(8-i) = \sum_{i=1}^{50} 8 - \sum_{i=1}^{50} i$$ *Using Theorem 2.1*

$$= 8(50) - \frac{50(50+1)}{2}$$

$$= 400 - 25(51)$$

$$= -875$$

Title:5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Example

2.5 Page: 335

Compute: $a)\ \sum_{i=1}^{20} i^2$ $b)\ \sum_{i=1}^{20} \left(\frac{i}{20}\right)^2$

a) $\sum_{i=1}^{20} i^2$ *Using Theorem 2.1*

$$\sum_{i=1}^{20} i^2 = \frac{20(20+1)(2(20)+1)}{6}$$

$$= \frac{20(21)(41)}{6}$$

$$= 2870$$

b) $\sum_{i=1}^{20} \left(\frac{i}{20}\right)^2 = \sum_{i=1}^{20} \frac{i^2}{(20)^2}$ *Simplify*

Using Theorem 2.2

$$= \frac{1}{400}\sum_{i=1}^{20} i^2$$ *Using Theorem 2.1*

$$= \frac{1}{400}\sum_{i=1}^{20} i^2 = \frac{1}{400} \times \frac{20(20+1)(2(20)+1)}{6}$$

$$= \frac{1}{400} \times \frac{20(21)(41)}{6} = \frac{2870}{400} = 7.175$$

Title: 5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Exercise Q13 Page: 337

Use summation rules to compute the sum: $Q13)\ \sum_{n=1}^{100}(n^2-3n+2)$ $\quad Q)\ \sum_{i=7}^{20} 5$

Solution

Q13) $\sum_{n=1}^{100}(n^2-3n+2)$ *Using Theorem 2.2*

$$\sum_{n=1}^{100}(n^2-3n+2)=\sum_{n=1}^{100}n^2-3\sum_{n=1}^{100}n+\sum_{n=1}^{100}2$$

Using Theorem 2.1

$$=\frac{100(100+1)(2(100)+1)}{6}-3\frac{100(100+1)}{2}+2(100)$$

$$=\frac{100(101)(201)}{6}-3(50)(101)+200$$

$$=338350-15150+200=323,400$$

Q) $\sum_{i=7}^{20} 5$

Here, we start from $i=7$

$$\sum_{i=7}^{20}5=\sum_{i=1}^{20}5-\sum_{i=1}^{6}5$$

Using Theorem 2.1

$$=5(20)-5(6)=70$$

Or

$$\sum_{i=7}^{20}5=5(20-7+1)=70$$

Number of terms

Title: 5.2 Sums and Sigma Notation

Lesson Objective: Use the sigma notation to compute basic summation.

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Example Use summation rules to compute the sum: $Q16)\ \sum_{i=4}^{20}(i-3)(i+3)$

Q16 Page: 337

Q16) $\sum_{i=4}^{20}(i-3)(i+3)$ *Multiply brackets*

$$\sum_{i=4}^{20}(i^2-3i+3i-9)=\sum_{i=4}^{20}(i^2-9)$$

Using Theorem 2.2

$$\sum_{i=4}^{20}(i^2-9)=\sum_{i=4}^{20}i^2-\sum_{i=4}^{20}9$$

Here, we start from $i=4$

$$\sum_{i=4}^{20}(i^2-9)=\sum_{i=1}^{20}i^2-\sum_{i=1}^{3}i^2-\sum_{i=4}^{20}9$$

Using Theorem 2.1

$$=\frac{20(20+1)(2(20)+1)}{6}-\frac{3(3+1)(2(3)+1)}{6}-9(20-4+1)$$

Number of terms

$$=\frac{20(21)(41)}{6}-\frac{3(4)(7)}{6}-9(17)$$

$$=2,870-14-153=2703$$

Lesson Objective: **Use the sigma notation to compute basic summation.**

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Exercise

Q17 Page: 337

Use summation rules to compute the sum: $Q17)\ \sum_{k=3}^{n}(k^2-3)$

Q16) $\sum_{k=3}^{n}(k^2-3)$

Using Theorem 2.2

$$\sum_{k=3}^{n}(k^2-3)=\sum_{k=3}^{n}k^2-\sum_{k=3}^{n}3$$

Here, we start from $k=3$

$$\sum_{k=3}^{n}(k^2-3)=\sum_{k=1}^{n}k^2-\sum_{k=1}^{2}k^2-\sum_{k=3}^{n}3$$

Using Theorem 2.1

$$=\frac{n(n+1)(2(n)+1)}{6}-(1^2+2^2)\quad-3(n-3+1)$$

Number of terms

$$=\frac{n(n+1)(2n+1)}{6}-5\quad-3(n-2)$$

$$=\frac{n(n+1)(2n+1)}{6}-5-3n+6$$

$$=\frac{n(n+1)(2n+1)}{6}-3n+1$$

Exercise

Use summation rules to compute the sum: $Q18)\ \sum_{k=0}^{n}(k^2+5)$

Q18 Page: 337

Q16) $\sum_{k=0}^{n}(k^2+5)$

Using Theorem 2.2

$$\sum_{k=0}^{n}(k^2+5)=\sum_{k=0}^{n}k^2+\sum_{k=0}^{n}5$$

Here, we start from $k=0$

$$\sum_{k=0}^{n}(k^2+5)=0^2+\sum_{k=1}^{n}k^2+\sum_{k=0}^{n}5$$

When $k=0$

Using Theorem 2.1

$$=0+\frac{n(n+1)(2(n)+1)}{6}+5(n-0+1)$$

Number of terms

$$=\frac{n(n+1)(2n+1)}{6}+5(n+1)$$

$$=\frac{n(n+1)(2n+1)}{6}+5n+5$$

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Example

Compute the sum and the limit of the sum as $n \to \infty$: $\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$

Q23 Page: 338

$$\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$$

Using Theorem 2.2

$$\frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right] = \frac{1}{n}\left[\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2 + 2\sum_{i=1}^{n}\left(\frac{i}{n}\right)\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n}\frac{i^2}{n^2} + 2\sum_{i=1}^{n}\left(\frac{i}{n}\right)\right] = \frac{1}{n}\left[\frac{1}{n^2}\sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} i\right]$$

$$= \frac{1}{n^3}\sum_{i=1}^{n} i^2 + \frac{2}{n^2}\sum_{i=1}^{n} i$$

Using Theorem 2.1

$$= \frac{1}{n^3} \times \frac{n(n+1)(2(n)+1)}{6} + \frac{2}{n^2} \times \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)}{6n^3} + \frac{n(n+1)}{n^2} \quad = \frac{(n+1)(2n+1)}{6n^2} + \frac{(n+1)}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right] = \lim_{n\to\infty}\left(\frac{(n+1)(2n+1)}{6n^2} + \frac{(n+1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2} + \lim_{n\to\infty}\frac{n+1}{n}$$

$$= \lim_{n\to\infty}\frac{2n^2+3n+1}{6n^2} + \lim_{n\to\infty}\frac{n+1}{n}$$

Highest degree term in numerator and denominator

$$= \lim_{n\to\infty}\frac{2n^2}{6n^2} + \lim_{n\to\infty}\frac{1n}{1n} \quad = \frac{2}{6} + 1 \quad = \frac{4}{3}$$

Lesson Objective: **Use the sigma notation to compute basic summation.**

Class & Safety rule	Warm Up Focus Quest.	Learning Obj. Key words	Starter Activity	Lesson Explanation	Main Activity	Students Involvement	Work evaluation	Plenary question	Self assessment	Enrichment

Exercise

Q25 Page: 338

Compute the sum and the limit of the sum as $n \to \infty$: $\sum_{i=1}^{n} \frac{1}{n}\left[4\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right)\right]$

$$\sum_{i=1}^{n} \frac{1}{n}\left[4\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right)\right]$$

Using Theorem 2.2

$$\frac{1}{n}\sum_{i=1}^{n}\left[4\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right)\right] = \frac{1}{n}\left[\sum_{i=1}^{n} 4\left(\frac{2i}{n}\right)^2 - \sum_{i=1}^{n}\left(\frac{2i}{n}\right)\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n}\frac{16i^2}{n^2} - \sum_{i=1}^{n}\left(\frac{2i}{n}\right)\right] = \frac{1}{n}\left[\frac{16}{n^2}\sum_{i=1}^{n} i^2 - \frac{2}{n}\sum_{i=1}^{n} i\right]$$

$$= \frac{16}{n^3}\sum_{i=1}^{n} i^2 - \frac{2}{n^2}\sum_{i=1}^{n} i$$

Using Theorem 2.1

$$= \frac{16}{n^3} \times \frac{n(n+1)(2(n)+1)}{6} - \frac{2}{n^2} \times \frac{n(n+1)}{2}$$

$$= \frac{16n(n+1)(2n+1)}{6n^3} - \frac{2n(n+1)}{2n^2} = \frac{8(n+1)(2n+1)}{3n^2} - \frac{(n+1)}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} \frac{1}{n}\left[4\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right)\right] = \lim_{n\to\infty}\left(\frac{8(n+1)(2n+1)}{3n^2} - \frac{(n+1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{8(n+1)(2n+1)}{3n^2} - \lim_{n\to\infty}\frac{n+1}{n}$$

$$= \lim_{n\to\infty}\frac{16n^2+24n+8}{3n^2} - \lim_{n\to\infty}\frac{n+1}{n}$$

Highest degree term in numerator and denominator

$$= \lim_{n\to\infty}\frac{16n^2}{3n^2} - \lim_{n\to\infty}\frac{1n}{1n} = \frac{16}{3} - \frac{1}{1} = \frac{13}{3}$$

Example 2.6 Page: 335

Sum the values of $f(x) = x^2 + 3$, evaluated at $x = 0.1, x = 0.2, x = 0.3, \dots, x = 1.0$

The terms are:

$$a_1 = f(0.1) = (0.1)^2 + 3$$

$$a_2 = f(0.2) = (0.2)^2 + 3$$ *And so, on*

x – *values are multiples of* 0.1

$x = 0.1,\quad x = 0.1(1)$

$x = 0.2,\quad x = 0.1(2)$

$x = 0.3,\quad x = 0.1(3)$

$\dots,$

$x = 1.0\quad x = 0.1(10)$

$1.0 = 0.1i$

$i = 10$

$x = 0.1\, i \qquad i = 1, 2, 3, \dots, 10$

$a_i = f(0.1i) = (0.1i)^2 + 3$ *For* $i = 1, 2, 3, \dots, 10$

$$\sum_{i=1}^{10} a_i = \sum_{i=1}^{10} f(0.1i) = \sum_{i=1}^{10} \left((0.1i)^2 + 3\right)$$ *Using Theorem 2.2*

$$= \sum_{i=1}^{10} (0.1)^2 i^2 + \sum_{i=1}^{10} 3 = (0.1)^2 \sum_{i=1}^{10} i^2 + \sum_{i=1}^{10} 3$$ *Using Theorem 2.1*

$$= 0.01 \times \frac{10(10+1)(2(10)+1)}{6} + 3(10)$$

$$= 0.01 \times \frac{10(11)(21)}{6} + 30$$

$$= 3.85 + 30 = 33.85$$

Lesson Objective: **Use the sigma notation to compute basic summation.**

Exercise 2.7 Page: 335

Sum the values of $f(x) = 3x^2 - 4x + 2$, evaluated at $x = 1.05, x = 1.15, x = 1.25, \dots, x = 2.95$

Solution *The values of x are:*

$i = 1$ $\quad i = 2$ $\quad i = 3, \dots$

$x = 1.05, \quad 1.15, \quad 1.25, \dots, 2.95$

$+0.1 \quad +0.1$

x – *values are increasing by* 0.1

$$x = 0.1\,i + c$$

For $i = 1$ $\quad 1.05 = 0.1(1) + c$

$$c = 1.05 - 0.1 = 0.95$$

$$x = 0.1\,i + 0.95$$

The last value of x $\quad 2.95 = 0.1i + 0.95$

$$2.95 - 0.95 = 0.1i$$

$2 = 0.1\,i \Rightarrow i = 20$

$x = 0.1\,i + 0.95 \quad i = 1, 2, 3, \dots, 20$

$a_i = f(0.1i + 0.95) = 3(0.1i + 0.95)^2 - 4(0.1i + 0.95) + 2$

For $i = 1, 2, 3, \dots, 20$

$$\sum_{i=1}^{20} a_i = \sum_{i=1}^{20} f(0.1i + 0.95)$$

$$= \sum_{i=1}^{20} \left(3(0.1i + 0.95)^2 - 4(0.1i + 0.95) + 2\right)$$

You can use calculator to find the sum directly

$$= \sum_{i=1}^{20} \left(3\left(0.01i^2 + 2(0.1)(0.95)i + 0.9025\right) - 0.4i - 3.8 + 2\right)$$

Exercise 2.7 Page: 335

Sum the values of $f(x) = 3x^2 - 4x + 2$, evaluated at $x = 1.05, x = 1.15, x = 1.25, \ldots, x = 2.95$

Solution

$$= \sum_{i=1}^{20} (3(0.01i^2 + 2(0.1)(0.95)i + 0.9025) - 0.4i - 3.8 + 2)$$

$$= \sum_{i=1}^{20} (0.03i^2 + 0.57i + 2.7075 - 0.4i - 1.8)$$

$$= \sum_{i=1}^{20} (0.03i^2 + 0.17i + 0.9075)$$ *Using Theorem 2.2*

$$= 0.03 \sum_{i=1}^{20} i^2 + 0.17 \sum_{i=1}^{20} i + \sum_{i=1}^{20} 0.9075$$ *Using Theorem 2.1*

$$= 0.03 \times \frac{20(20+1)(2(20)+1)}{6} + 0.17 \times \frac{20(20+1)}{2} + 0.9075(20)$$

$$= 0.03 \times \frac{20(21)(41)}{6} + 0.17 \times \frac{20(21)}{2} + 18.15$$

$$= 86.1 + 35.7 + 18.15 \quad = 139.95$$

Title: 5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Example Compute the sum of the form $\sum_{i=1}^{n} f(x_i)\Delta x$ for the given values of x_i

$$f(x) = x^2 + 4x, \qquad x = 0.2, 0.4, 0.6, 0.8, 1.0 \qquad \Delta x = 0.2; n = 5$$

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Solution *The values of x_i are:*

$x_i = 0.2, \quad 0.4, \quad 0.6, \quad 0.8, \quad 1.0$

$\Delta x = 0.2$

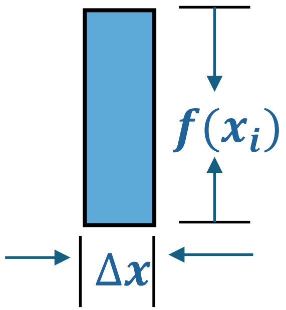
x *– values are multiples of* 0.2

$x_i = 0.2i$ $\qquad i = 1, 2, 3, \ldots\ldots, n$

$\qquad i = 1, 2, 3, \ldots\ldots, 5$

$f(x_i)\Delta x$

Represent the area of a rectangle of
Length= $f(x_i)$ width= Δx



$\sum_{i=1}^{n} f(x_i)\Delta x$

Represent the estimated area under graph of $f(x)$ on [0.2,1.0]

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{5} f(0.2i)\,(0.2)$$

$$= \sum_{i=1}^{5}\left((0.2i)^2 + 4(0.2i)\right)0.2 = \sum_{i=1}^{5}\left((0.2)^3 i^2 + 0.16i\right)$$

Using Theorem 2.2

$$= (0.2)^3\sum_{i=1}^{5} i^2 + 0.16\sum_{i=1}^{5} i$$ *Using Theorem 2.1*

$$= 0.008 \times \frac{5(5+1)(2(5)+1)}{6} + 0.16 \times \frac{5(5+1)}{2}$$

$$= 0.008 \times \frac{5(6)(11)}{6} + 0.16 \times \frac{5(6)}{2}$$

$$= 0.008 \times 5(11) + 0.16 \times 5(3)$$

$$= 0.008 \times 55 + 0.16 \times 15 = 2.84$$

Exercise Compute the sum of the form $\sum_{i=1}^{n} f(x_i)\Delta x$ for the given values of x_i **Q21 Page: 337**

$$f(x) = 4x^2 - 2, \qquad x = 2.1, 2.2, 2.3, 2.4, \ldots\ldots 3.0 \;\; \Delta x = 0.1;\; n = 10$$

Solution *The values of x_i are:*

$x_i = 2.1, \quad 2.2, \quad 2.3, \quad 2.4, \ldots\ldots\ldots 3.0$

$\Delta x = 0.1$ $\quad$ *x − values are increasing by* 0.1

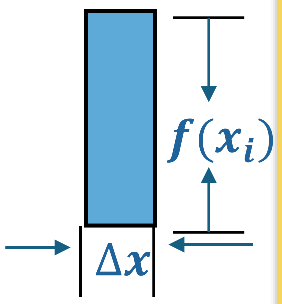
$x_i = 0.1i + c$ $\quad$ $i = 1, 2, 3, \ldots\ldots, n$

$i = 1, 2, 3, \ldots\ldots, 10$

For $i = 1$ $\quad 2.1 = 0.1(1) + c$

$c = 2.1 - 0.1 = 2$

$x_i = 0.1\,i + 2$



$f(x_i)\Delta x$ *Represent the area of a rectangle of Length= $f(x_i)$ width= Δx*

$\sum_{i=1}^{n} f(x_i)\Delta x$ *Represent the estimated area under graph of $f(x)$ on* [2.1,3.0]

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{10} f(0.1i + 2)(0.1) = \sum_{i=1}^{10} (4(0.1i + 2)^2 - 2)0.1$$

$$= \sum_{i=1}^{10} (4(0.01i^2 + 0.4i + 4) - 2)0.1$$

$$= \sum_{i=1}^{10} (0.04i^2 + 1.6i + 16 - 2)0.1 = \sum_{i=1}^{10} (0.004i^2 + 0.16i + 1.4)$$

Using Theorem 2.2

$$= 0.004\sum_{i=1}^{10} i^2 + 0.16\sum_{i=1}^{10} i + \sum_{i=1}^{10} 1.4$$

Using Theorem 2.1

$$= 0.004 \times \frac{10(10+1)(2(10)+1)}{6} + 0.16 \times \frac{10(10+1)}{2} + 1.4(10)$$

$$= 0.004 \times \frac{10(11)(21)}{6} + 0.16 \times \frac{10(11)}{2} + 14$$

$$= 24.34$$

ALEF

Title:5.2 Sums and Sigma Notation

Lesson Objective: **Use the sigma notation to compute basic summation.**

Thank You